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APPLICATION OF THE "RIGID STREAM TUBE" METHOD TO CALCULATE MICELLE-POLYMER FLOODING IN A STAGGERED SYSTEM OF WELLS

N. A. Inogamov[a] and N. S. Khabeev[b]

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The applicability of the simplified "rigid stream tube" method to many practical situations of calculating micelle-polymer flooding in a system of wells is shown. The scheme of motion of the oil fringe in the pool and the influence of the main parameters on the efficiency of the considered method for increasing oil recovery are analyzed.

Introduction. One of the most important and urgent problems is increasing oil production from the bowels of the earth. In obtaining oil from collectors by artificial flooding, it is impossible to extract it completely — usually no more than 50% of the initial stock is displaced. The main reason for the insufficient efficiency of this production process is oil confinement in a porous medium by capillary forces mainly due to the high surface tension at oil-water interfaces. On the basis of experimental and theoretical investigations it has been found that a more complete working of oil fields can be attained by decreasing considerably the surface tension between the displacing and pool liquids. According to the results of the laboratory and experimental studies of the physicochemical methods for increasing oil recovery, micelle-polymer flooding of oil pools is promising.

Basic Equations. To investigate the dynamics of the process of working some oil fields and determine the sought technological indices, the use of the so-called "rigid stream tube" method [1–4] proved to be convenient and efficient, although the filtration process of micelle-polymer displacement in two-dimensional regions is nonstationary and the rate of motion of liquid particles along different streamlines may differ considerably [5]. Let us consider the application of a variant of the method to calculate the flooding of a five-point scheme of oil pool working. Assuming the necessary homogeneity of the pool, let us investigate one-fourth of a symmetry element, which we will break up into N_t "stream tubes," i.e., subregions whose boundaries, as is considered, are the real streamlines of the filtering liquid (Fig. 1). Assume N_t to be an integer. In so doing, either volumes (areas) of the tubes or injection well angles can be chosen to be equal. For the condition of equal volumes of the tubes V_t, we have the following relations defining the values of the tube apex angles at the injection ($\delta\gamma_j^1$) and production ($\delta\gamma_j^2$) wells characterizing such stream tubes [4] (Fig. 1):

$$V_t = \mathrm{AB}\cdot\mathrm{AD}\,\frac{1}{N_t} = \text{const}\,;\quad \gamma_j^i = \arctan\,\frac{2j}{N_t}\left(\frac{\mathrm{AD}}{\mathrm{AB}}\right)^{-2i+3},\quad j = \overline{1, \frac{N_t}{2}},\quad i = 1, 2\,;\quad \delta\gamma_1^i = \gamma_1^i\,;$$
$$\delta\gamma_j^i = \gamma_j^i - \gamma_{j-1}^i\,,\quad j = \overline{2, \frac{N_t}{2}}\,;\quad \delta\gamma_j^i = \delta\gamma_{N_t-j+1}^{-i+3}\,,\quad j = \overline{\frac{N_t}{2} + 1, N_t}\,. \tag{1}$$

Let us assume that as a result of the proposed break-up real stream tubes for the filtration flow under consideration take place. Thus, independent flow of the mixture in each stream tube is obtained. Assume now that the flow in an arbitrary jth tube consisting of $\Delta\mathrm{AE}_j\mathrm{E}_{j+1}$ and $\Delta\mathrm{CE}_j\mathrm{E}_{j+1}$ is plane-parallel and depends only on one spatial coordinate — the radius. Let us replace triangles by sectors of adequate area with the same well angles. Then the radii of these sectors R_j^i are defined as follows:

[a]Institute of Mechanics, M. V. Lomonosov Moscow State University, 1 Michurnskii Ave., Moscow, 117234; [b]University of Bahrain, 32038, Kingdom of Bahrain. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 80, No. 1, pp. 15–21, January–February, 2007. Original article submitted August 2, 2006.

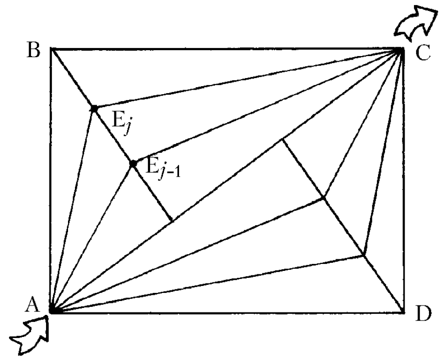

Fig. 1. Scheme of breaking the pool symmetry elements ABCD into N_t "stream tubes."

$$R_j^i = \frac{(2-i)\,\mathrm{AD}^2 + (i-1)\,\mathrm{CD}^2}{\mathrm{AC}} \sqrt{\frac{\sin \delta\gamma_j^i}{\delta\gamma_j^i \cos \gamma_j^i \cos \gamma_{j-1}^i}}\,, \quad j = \overline{1, \frac{N_t}{2}}\,; \tag{2}$$

$$R_j^i = R_{N_t - j + 1}^{-i+3}\,, \quad j = \overline{\frac{N_t}{2} + 1, N_t}\,, \quad i = 1, 2\,.$$

Here the superscript i indicates belonging to $\Delta \mathrm{AE}_j\mathrm{E}_{j+1}$ (i = 1) or $\Delta \mathrm{CE}_j\mathrm{E}_{j+1}$ (i = 2). The flow throughout the tube is considered to be continuous — equality of the rates of flow through the sector boundaries is assumed, while, generally speaking, $R_j^1 \delta\gamma_j^1 \neq R_j^2 \delta\gamma_j^2$. According to the terminology taken in [4], in the first sector with a center in the injection well the axially symmetric flow will be intra-contour and in the second one — extra-contour.

We will assume that at a given injection rate $Q(t)$ the volume flow of the liquid filtering in the jth tube is proportional to the tube opening $\delta\gamma_j^1$:

$$Q_j = Q\,(t)\,\frac{\delta\gamma_j^1}{2\pi}\,. \tag{3}$$

In the case of micelle-polymer flooding, in the five-point system at such a choice of rigid tubes the quantity of micella solution, as well as of thickened water injected in equal times into different tubes, will be different. It can be shown that the flow region cannot be broken up into triangular stream tubes so that the oil fringes injected into the tubes are equal with respect to the pore volumes of the tubes. Therefore, it is necessary to perform numerical calculations in each tube separately. The mathematical description of the plane-parallel flow model is given in [5]. To find an approximate solution of the considered problem, we assumed the following.

To take into account the pressure distribution on the well, let us single out circles concentric with the well contours, inside which the flow in sectors with a small opening will be assumed to be plane-parallel. Then the input equations in this region — sectors with angle $\delta\gamma_n$ — will take a simpler one-dimensional form:

$$\frac{\partial}{\partial r} q_n = 0\,, \quad \frac{\partial s}{\partial t} + \chi \frac{\partial F}{\partial r} = 0\,, \quad \frac{\partial\,(s_\alpha c_{\alpha,\beta} + a)}{\partial t} + \chi \frac{\partial F_\alpha c_{\alpha,\beta}}{\partial r} = 0\,, \quad \chi = \frac{q_n}{2\pi r}\,, \quad \alpha = 1\,, \quad \beta = 2, 3\,;$$
$$\alpha = 2\,, \quad \beta = 4\,, \quad q_n = -\,\delta\gamma_n r \Lambda \frac{\partial p}{\partial r}\,, \quad r \cup (r_w, r_*)\,, \quad \gamma \cup (\gamma_n, \gamma_{n+1})\,, \quad n = \overline{1, N_t}\,, \quad \delta\gamma_n = \gamma_{n+1} - \gamma_n\,. \tag{4}$$

On a singled out circle of radius r_*, the solution and the normal component of the volume flow are considered to be continuous, i.e., the problems in the "internal" and "external" regions, into which the filtration field is divided, are solved simultaneously, and their solutions and derivatives "coalesce" at the interface. In the sectors acting as a stream tube, the equations are of the form

$$\frac{\partial}{\partial t}\mathbf{V} + \chi\frac{\partial}{\partial r}\mathbf{W} = 0\,, \quad \mathbf{V} = \begin{pmatrix} s \\ sc_{1,3} \\ s_2 c_{2,4} \end{pmatrix}, \quad \mathbf{W} = \begin{pmatrix} F \\ Fc_{1,3} \\ (1-F)\, c_{2,4} \end{pmatrix}. \tag{5}$$

The substitution

$$\chi = (-1)^{i+1}\frac{Q_j}{Hr\delta\gamma_j^i}\,, \quad r \in \left(r_\mathrm{w}, R_j^i\right), \quad x^i = \left(r^2 - r_\mathrm{w}^2\right)\delta\gamma_j^i/(2V_\mathrm{t})\,, \tag{6}$$

and then substitution of x^2 by $1-x^2$ reduce the system of equations (5) to a form that admits the possibility of through counting along the full length of the tube:

$$\frac{\partial}{\partial t}\mathbf{V} + \chi\frac{\partial}{\partial x}\mathbf{W} = 0\,, \quad \chi = \frac{Q\delta\gamma_j^1}{2\pi HV_\mathrm{t}}\,, \quad x \in (0, 1)\,. \tag{7}$$

Knowing the solution of the system of equations (7) $s(x, t)$, $c_{1,3}(x, t)$, $c_{2,4}(x, t)$ in each tube, one can determine in them the current pressure distribution. In this case, the equation defining it

$$\nabla \sum_{\alpha} \Lambda_\alpha \nabla p = 0 \tag{8}$$

is of the form

$$\frac{Q}{2\pi Hr}\frac{\delta\gamma_j^1}{\delta\gamma_j^i} = (-1)^i\,\Lambda\frac{\partial p}{\partial r}\,, \quad r \in \left(r_\mathrm{w}, R_j^i\right), \quad i = 1, 2\,. \tag{9}$$

Let us carry out in (7) substitution of variables (6). Integrating the equation obtained, we have

$$P(x, t) = \begin{cases} -\chi_1 \displaystyle\int_0^x \frac{d\xi}{\Lambda\,(\chi_3\xi + \chi_5)}\,, \quad 0 \le x \le x_j^1\,; \\ P(x_j^1, t) - \chi_2 \displaystyle\int_{1-x_j^2}^{x} \frac{d\xi}{\Lambda\,(\chi_4\,(1-x) + \chi_5)}\,, \end{cases} \tag{10}$$

$$x \in [x_j^1, 1], \tau = \frac{\int_0^t Q(t)\,dt}{V_\mathrm{d}}\,, \quad \chi_i = \frac{Q\delta\gamma_j^1}{2\pi H\delta\gamma_j^i}\,, \quad \chi_{2+i} = \frac{2}{\delta\gamma_j^i}\,, \quad i = 1, 2\,, \quad \chi_5 = \frac{r_\mathrm{w}^2}{V_\mathrm{t}}\,,$$

where V_d is the dynamic volume of the porous medium occupied by a mobile liquid, and we assume thereby that $\sum_{i=1}^{2} x_j^i = 1$, i.e., we ignore the quantity $r_\mathrm{w}^2 \sum_{i=1}^{2} \delta\gamma_j^i 2N_\mathrm{t}/V_\mathrm{p}$.

Results of the Numerical Calculations. To satisfy the required stability condition of the difference scheme, in each tube the ratio between time and space steps should satisfy the Courant condition

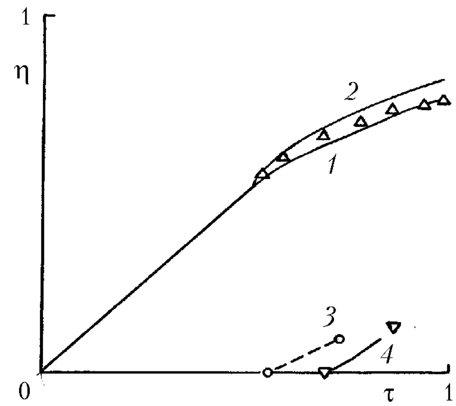

Fig. 2. Comparison of the results of the numerical calculations (1) to the experimental data of [6] (2) on current oil recovery in five-point micelle-polymer flooding; moments of water (3) and polymer (4) inrush into the production well.

$$\frac{\Delta t}{\Delta x} \le \frac{\pi}{2N_t \Delta\gamma_j^1 \max F'} . \quad (11)$$

So the calculations were performed with regard for the characteristic speeds of filtration in each stream tube. We checked the required condition of correctness of the calculations performed — observance of material balance in the jth stream tube:

$$- V_j \left(1 + \int_0^1 s\,(x, t)\, dx \right) = Q_j \left(1 - \int_0^t F\,(1, t)\, dt \right) . \quad (12)$$

The integrals in (10) were calculated by the Simpson method. In the vicinity of the injection well, where the integral has a singularity, it was calculated analytically with the use of the mean-value theorem

$$p\,(\varepsilon, \tau) = \frac{-Q}{4\pi k H} \frac{\mu_2}{k_2\,(\Delta x, \tau)} \ln \frac{\varepsilon + x_0}{x_0} . \quad (13)$$

In the calculations, $\varepsilon = 2\Delta x$ was assumed.

We simulated numerically the experiment [6], in which, as a pool model, a Bury bulk sandstone parallelepiped with dimensions 44.5:62:1.5 cm and porosity $m = 0.2$ was used. The absolute permeability $k = 0.5 \mu m^2$. The pool was initially saturated with oil at a residual water saturation $s_0 = 0.827$. The viscosities of the pool oil (kerosene) and water are, respectively, 1.8 and 1 kPa·sec. The phase permeabilities of water and oil for the given porous medium are presented in [5, 7]. The composition of the micelle solution was as follows: $c_{1,1}^0 = 0.45$, $c_{1,2}^0 = 0.34$, $c_{1,3}^0 = 0.21$. Contacting the pool liquids, the solution formed a two-phase structure. The micelle solution viscosity relevant to the given composition is 21 kPa·sec.

The buffer liquid was modeled by a polyacrylamide solution with a fictitious viscosity of 28 kPa·sec. The sorption of surface-active substances was insignificant, and the sorption isotherm in the calculations was taken in the form of the Henry law (H = 0.3). The calculations were performed for the dynamic variables for $s_1^* = 0$.

The numerical solution of the system of equations (7) with the above-described closing relations has been determined under the following initial and boundary conditions:

$$\tau = 0 , \ \ \bar{s}_0 = 1 , \ \ \bar{c}_{1,3} = \bar{c}_{2,4} = 0 ;$$

$$\tau \cup (0, t_1) , \ \ \bar{s}\,(0, \tau) = 1 , \ \ \bar{c}_{1,3} = 1 , \ \ \bar{c}_{2,4} = 0 ;$$

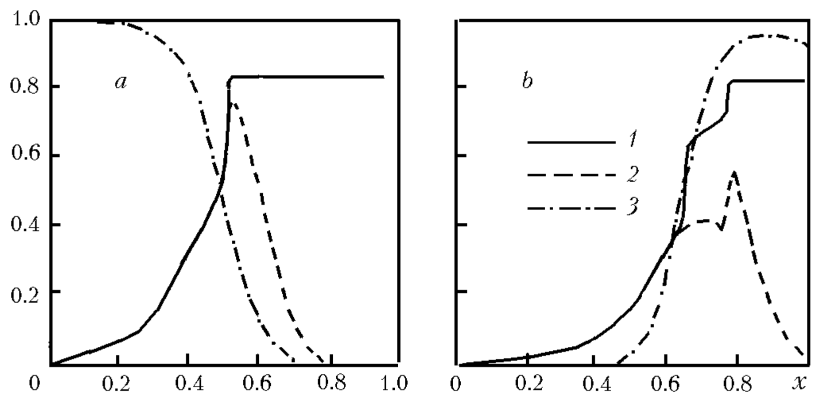

Fig. 3. Distributions of the saturation with the hydrocarbon phase (1), and SAS (2) and polymer (3) concentrations in "stream tube" 4 at various instants of time: a) τ = 0.3; b) 0.4.

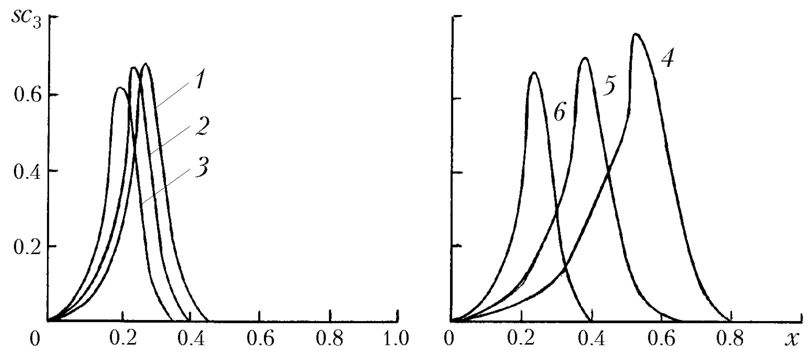

Fig. 4. Distribution of micelle solution fringes in different stream tubes at time τ = 0.3; 1–6, stream tube numbers.

$$\tau \cup (t_1, t_2)\,, \quad \bar{s}\,(0, \tau) = 0\,, \quad \bar{c}_{1,3} = 0\,, \quad \bar{c}_{2,4} = 1\,; \tag{14}$$

$$\tau \cup (t_2, t)\,, \quad \bar{s} = 0\,, \quad \bar{c}_{1,3} = 0\,, \quad \bar{c}_{2,4} = 0\,.$$

Here τ = 0.1 and 0.6 are, respectively, the time of injection of the micelle solution into the pool and the time of termination of injection of the buffer liquid. The number of stream tubes N_t, into which 1/4th part of the pool element is broken up, was taken to be equal to 6.

Figure 2 compares the results of the numerical calculations with those of the experimental investigations on the dynamics of water-oil recovery in a five-point micelle-polymer flooding. Figure 2 shows the dependence of the current oil recovery related to the pore volume of the pool:

$$\eta = \sum_{j=1}^{N_t} \frac{Q_j}{V_j} \int_0^{\tau} F\,(1, \tau)\,(1 - c_{1,3}\,(1, \tau))\,d\tau\,, \tag{15}$$

on the dimensionless volume of pumped liquid. As is seen, the oil is displaced incompletely (the average oil saturation in the pool upon injection of one V_p was 7.8%).

The numerical solution, correctly expressing the dynamics of water and buffer liquid inrush into the production well, shows agreement with the experimental data.

Figure 3 shows the character of the time evolution of the distribution of the oil-saturation and concentration of surface active substances (SAS) in the tubes with the example of the 4th stream tube. It is seen that the mechanism of oil displacement is similar to the linear case, i.e., constraint of the micelle solution occurs and a water billow is

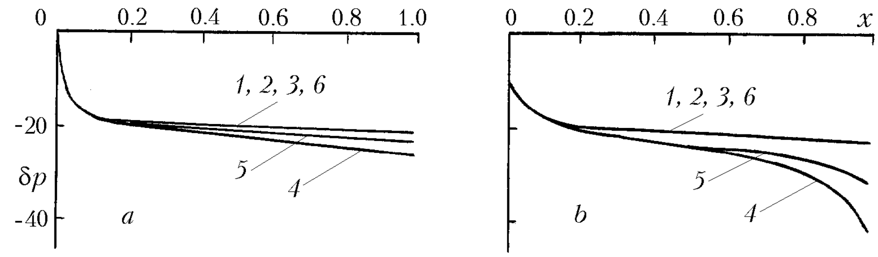

Fig. 5. Pressure distribution in different stream tubes at τ = 0.3 (a) and 0.6 (b): 1–6, stream tube numbers.

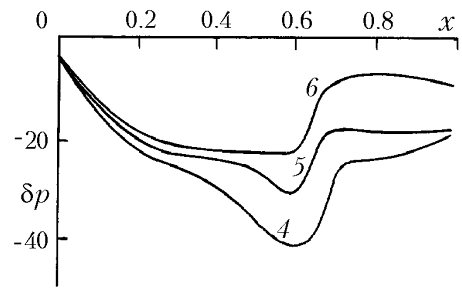

Fig. 6. Pressure drop between wells on stream tubes 4, 5, and 6.

formed in the region of the micelle solution because of the polymer adsorption. The absence of an oil billow is explained by the initial conditions. Approximately by the time τ = 0.5 the whole of the oil (in this tube) is displaced, the micelle solution rushes into the production well, and subsequently only the constrained solution remains. The burst on the SAS concentration curve for τ = 0.4 is due to the superposition of two jumps of the functions $c_{1,3}$ and s, which in numerical calculations are spread to a different extent, i.e., this is a limitation of the difference scheme rather than a characteristic feature of the solution. In the other tubes the mechanism of displacement is analogous to the considered one except for the time discrepancy and the size of injected oil fringes

$$V_{\mathrm{m.s}j} = \frac{V_{\mathrm{m.s}}}{2\pi}\,\delta\gamma_k^i\,, \quad k = \begin{cases} j,\ i = 2, j \le 3\,; \\ j - 3,\ i = 1, j > 3\,. \end{cases} \tag{16}$$

Figure 4 shows the oil fringe position in different stream tubes for τ = 0.3. It is seen that the oil fringes reach the tube end at different instants of time. Their volumes in the tubes also differ.

The pressure distribution in the stream tubes for the instants of time τ = 0.3 and 0.6 is given in Fig. 5. Before the break of an oil fringe and termination of injection in viscous liquids, the pressure drop between the tubes along the length is small. For τ = 0.6 the pressure distribution in the tubes differs especially near the production well because of the early break of the oil fringe in tubes 4 and 5.

The practical coincidence of the pressure drop curves points to the absence of large hydrodynamic pressure drops between the tubes, which is indicative of a good fit of the chosen rigid tubes to real stream tubes.

Figure 6 shows the pressure drop between the production and injection wells in stream tube 4. The increase and decrease in the pressure is due to the injection of high- and low-viscosity liquids, i.e., the pressure drops needed to hold the rate of pumping liquids into the pool constant are maximum when it simultaneously contains high-viscosity fringes of the micelle solution and the buffer liquid. Up to the instant of time τ = 0.6 (breakthrough of the micelle solution fringe) δp increases and after the beginning of injection of a low-viscosity water into the pool it decreases.

Conclusions. the possibility of using the "rigid stream tube" method to calculate micelle-polymer flooding in a system of wells has been analyzed. Comparison to the experimental data on the current oil recovery has been made. The scheme of motion of the oil fringe has been analyzed. The applicability of the "rigid stream tube" method to many practical situations has been shown.

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NOTATION

a, concentration of component adsorbed by the porous medium skeleton; $c_{\alpha,\beta}$, mass concentration of component in a phase; $c_{1,3}$, concentration of surface active substances in the hydrocarbon phase; $c_{2,4}$, concentration of polymer in the water phase; F, Buckley–Leverette function; F', partial derivative of F with respect to s; H, thickness of the pool; k, absolute permeability of the porous medium; k_α, relative phase permeability; m, medium porosity; N_t, number of stream tubes; p, pressure; Q, volume flow; $q = Q/H$, flow rate in the two-dimensional problem; R, radius of the well neighborhood; r, radial coordinate; r_*, singled-out radius; s_α, phase saturation; $s_1 = s$, oil saturation; $s_2 = 1 - s$, water saturation; t, time; Δt, time step of integration; V, tube volume; $V_{\mathrm{m.s}j}$, relative volume of the oil fringe injected into the jth tube; **V**, **W**, vectors; x, spatial coordinate; Δx, integration step for x; H, Henry constant; γ, angle; $\delta\gamma$, angle in the stream tube near the well; $\delta\gamma_j^1$, apex angle of the tube at the injection well; $\delta\gamma_j^2$, apex angle of the tube at the production well; δp, pressure drop; ε, radius of the small neighborhood around the well; η, current oil recovery coefficient; Λ, conductance of the mixture ($\Lambda = k\sum\limits_{\alpha=1}^{2}\frac{k_\alpha}{\mu_\alpha}$); μ, viscosity; ξ, integration variable; τ, dimensionless time (coincides with the dimensionless volume of injected liquid (with respect to the porous volume of the pool)); $\chi = q_n/2\pi r$. Subscripts: m.s, micelle solution; p, porous; w, well; d, dynamic; j, stream tube number; 0, initial value; α, phase index ($\alpha = 1$, hydrocarbon phase; $\alpha = 2$, water phase); β, component index ($\beta = 1$, oil; $\beta = 2$, water; $\beta = 3$, SAS; $\beta = 4$, polymer); overbar, dimensionless parameter (often omitted for simplification); *, remainder value; t, stream tube.

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